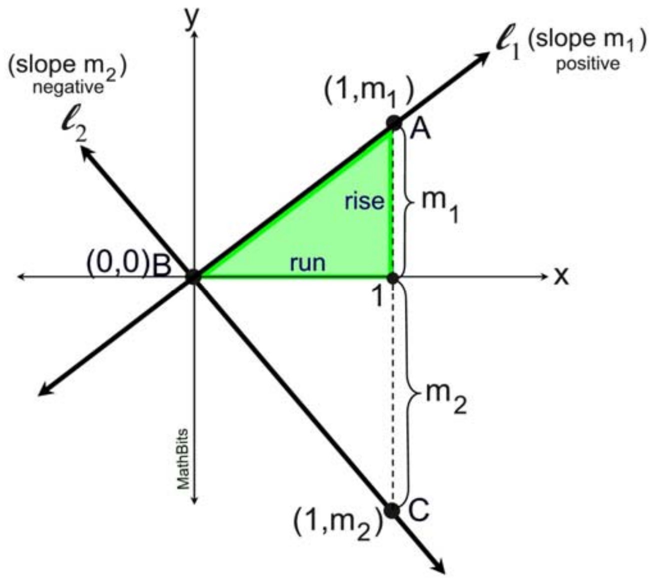
Slope Criteria for Perpendicular Lines

Name______________________________

Directions: Supply reasons.

Theorem: If two lines are perpendicular, their slopes are negative reciprocals. (The product of the slopes = -1.)



Given: $\ell_1 \perp \ell_2$ (let m_1 = slope of ℓ_1; m_2 = slope of ℓ_2)
Prove: $m_1 = -1/m_2$ ($m_1 \cdot m_2 = -1$)

For ease of computation, translate the perpendicular lines so the point of intersection will be the origin. Draw a vertical line $x = 1$ to form ΔABC.

We will be using the Distance Formula and the Pythagorean Theorem to complete this proof.

Proof:

1. $\ell_1 \perp \ell_2$ with vertical line $x = 1$ (let m_1 = slope of ℓ_1; m_2 = slope of ℓ_2)

2. The vertical line, $x = 1$, intersects ℓ_1 at $(1, m_1)$ and ℓ_2 at $(1, m_2)$.

3. $\angle ABC$ is a right angle.

4. ΔABC is a right triangle.

5. $BA = \sqrt{1+m_1^2}$

$BC = \sqrt{1+m_2^2}$

$AC = \sqrt{(1-1)^2+(m_1-m_2)^2} = m_1 - m_2$

6. $\left(\sqrt{1+m_1^2}\right)^2 + \left(\sqrt{1+m_2^2}\right)^2 = (m_1-m_2)^2$

7. $1+m_1^2+1+m_2^2 = (m_1-m_2)^2$

8. $2+m_1^2+m_2^2 = m_1^2 - 2m_1m_2 + m_2^2$

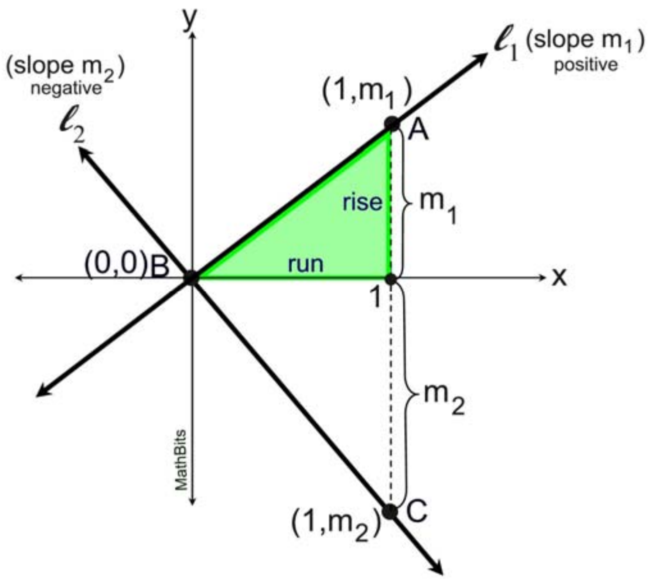
9. $2 = -2m_1m_2$

10. $-1 = m_1m_2$

11. $m_1 = -\dfrac{1}{m_2}$

Theorem: If the slopes of two lines are negative reciprocals, the lines are perpendicular. (The product of the slopes = -1.) (Let m_1 = slope of ℓ_1; m_2 = slope of ℓ_2)

Given: $m_1 = -1/m_2$ ($m_1 \cdot m_2 = -1$)
Prove: $\ell_1 \perp \ell_2$



For ease of computation, translate the lines so the point of intersection will be the origin. Draw a vertical line $x = 1$ to form ΔABC.

We will be using the Distance Formula to express the sides of ΔABC, and then we will attempt to show that the sides satisfy the Pythagorean Theorem proving ΔABC to be a right triangle (making the lines perpendicular).

Proof:

1. $m_1 = -1/m_2$; ($m_1 \cdot m_2 = -1$);
vertical line $x = 1$

2. The vertical line, $x = 1$, intersects ℓ_1 at $(1, m_1)$ and ℓ_2 at $(1, m_2)$.

3. $BA = \sqrt{1+m_1^2}$

$BC = \sqrt{1+m_2^2}$

$AC = \sqrt{(1-1)^2+(m_1-m_2)^2} = m_1 - m_2$

Will the sides of ΔABC satisfy the Pythagorean Theorem?

4. $\left(\sqrt{1+m_1^2}\right)^2+\left(\sqrt{1+m_2^2}\right)^2 \stackrel{?}{=} (m_1-m_2)^2$

5. $1+m_1^2+1+m_2^2 \stackrel{?}{=} (m_1-m_2)^2$

6. $2+m_1^2+m_2^2 \stackrel{?}{=} m_1^2-2m_1m_2+m_2^2$

7. $2 \stackrel{?}{=} -2m_1m_2$

8. $-1 \stackrel{?}{=} m_1m_2$

9. $m_1 = -\frac{1}{m_2}$

10. ΔABC is a right Δ.

11. $\angle ABC$ is right $\angle$.

12. $\ell_1 \perp \ell_2$