

# Communication and Mathematics

Name ANSWER KEY

(The Language of Mathematics - prime numbers)

**Will Aliens Really Speak Math???** In the movie “Contact”, author and astronomer Carl Sagan envisions communication with an alien race through the use of mathematics. A signal containing **prime numbers** is received from a system called Vega.

Today’s scientists support Carl Sagan’s hypothesis that any signal we receive from intelligent life is unlikely to be in English. They hypothesize that such a signal will be in the language of mathematics and that we listen for a signal containing the first few prime numbers: 2, 3, 5, 7, 11, 13, ... as an indication of intelligent life.



*Remember:* An integer greater than one is **prime** if its only positive divisors are itself and one (otherwise it is **composite**).

1. Complete this listing of the first 10 prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
2. The Greek mathematician, Eratosthenes (circa 200 BC), devised a “sieve” for finding prime numbers, called the Sieve of Eratosthenes. The sieve sifts out composite numbers and leaves prime numbers behind. Use the sieve to find the prime numbers less than 100 in the chart below.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

### To use the sieve:

- Cross out 1, because it is not prime.
- Circle 2, as it is the smallest positive even prime. Cross out every multiple of 2 (every second number).
- Circle 3, the next prime. Cross out all of the multiples of 3 (every third number). Some may already be crossed out as multiples of 2.
- Circle the next available number, 5. Cross out all of the multiples of 5 (every fifth number).
- Continue this process until all numbers through 100 have either been circled or crossed out.
- The circled numbers are the prime numbers from 1 to 100

3. How many prime numbers are between 1 and 100? 25

The *Fundamental Theorem of Arithmetic* states that every positive integer is a product of prime numbers in one and only one way (not considering the order of the factors).

4. Write 48 as a product of its prime factors.  $3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 3 \cdot 2^4$
5. Find all of the numbers less than 50 that have only 2 prime factors. For example,  $12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$  is considered to be a number with only 2 prime factors.

12, 15, 18, 20, 21, 22, 24, 26, 28, 33, 34, 35, 38, 39, 40, 44, 46, 48

6. *Twin* primes are prime numbers that differ by two, such as 3 and 5. Find 3 other sets of twin primes. 5, 7, 11, 13, 17, 19 and others
7. Mathematician Sophie Germain discovered a relationship between certain prime numbers. A prime number  $p$  is called a “*Germain prime*” if  $2p + 1$  is also prime. List the first 5 Germain primes.  
2, 3, 5, 11, 23
8. Two natural numbers are called *relatively* prime if they have no common factor greater than 1. Find two relatively prime natural numbers that are each greater than 50. 51, 52
9. Mathematicians remain stumped by the task of predicting when the next prime will occur. It is suspected that there are no patterns for predicting prime numbers. Since primes remain such an enigma, they are often referred to as mathematics’ lucky numbers.

Choose a prime number less than 50 (your favorite one) answers will vary  
List 5 situations where you have seen, or heard of, the occurrence of this number (such as part of your locker number, your favorite TV channel, in a song, etc.)

---



---



---



---



---

E-commerce revels in the fact that we truly do not understand prime numbers. Every time a credit card is used on the internet to buy an item, the account is kept secret from hackers by the power of prime numbers. An e-business chooses two large primes,  $p$  and  $q$ , which are kept secret. A calculation using  $P$  (*the product of  $p$  and  $q$* ) encrypts the credit card. Any attempt to decode the information would need knowledge of the secret primes  $p$  and  $q$ . Mathematicians lack any fast way of cracking a huge number,  $P$ , into its prime factors.

10. Find the missing prime number in this encryption equation.  
Encryption =  $A^2 + 2.71(B) - 3.14(C)$ , where  $A$ ,  $B$  and  $C$  are prime.  
Encryption = 334.21  
 $A = 17$   
 $C = 2$   
 $B = \underline{19}$

11. Find the missing prime number in this encryption equation.  
Encryption =  $A^3 + 2.46B - C^{-2}$ , where  $A$ ,  $B$  and  $C$  are prime.  
Encryption = 1362.73  
 $A = 11$   
 $B = 13$   
 $C = \underline{2}$