Infinite Series

Zeno's Paradox – *I.Q.*

Name____**ANSWER KEY**_____

Zeno's Paradox: In the movie *I.Q.*, Meg Ryan wishes to cross the room to Tim Robbins. First, she must cover half of the distance. Then, she must cover half of the remaining distance. Then, she must cover half of the remaining distance. Then she must cover half of the remaining distance, and so on forever. The implication of this theory is that she will never reach Tim Robbins, thus making all motion impossible since it involves moving an infinite number of small intermediate distances first. We know this is crazy – but it sounds logical, doesn't it? Let's look further.

Consider: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots$ This infinite series can also be written as $\sum_{n=1}^{\infty}\frac{1}{2^n}$.

Examine the partial sums: $\left(\frac{1}{2},\frac{3}{4},\frac{7}{8},\frac{15}{16},\frac{31}{32},\ldots\right)$ The limit of this sequence is **1.**

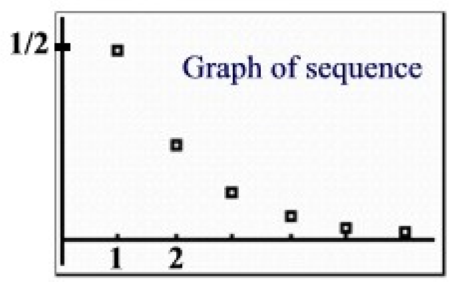
> *** The **sum of an infinite series** is the limit of the sequence of partial sums. If this limit has a finite value, the series is said to *converge*. If not, the series is said to *diverge*.

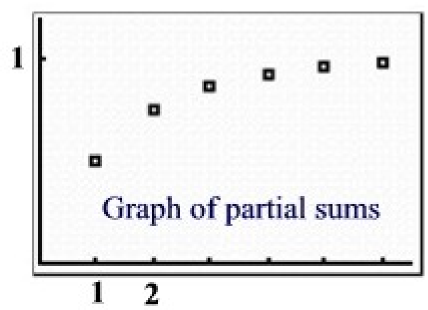
The fact that an infinite series can converge resolves Zeno's paradox.

$1=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots$ The total distance between Meg and Tim, represented by 1, is really the addition of all the "half" distance interludes. Happily she CAN walk across the room!!

Stat Plot on calculator:

Compare the graph of the sequence $\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16},\frac{1}{32},\ldots$ and the graph of the partial sums.





Exercises: *For each series presented, determine if the series converges or diverges. List at least the first 5 partial sums for each series. If the series converges, state the value. Show graphical support for your conclusions. Please label.*

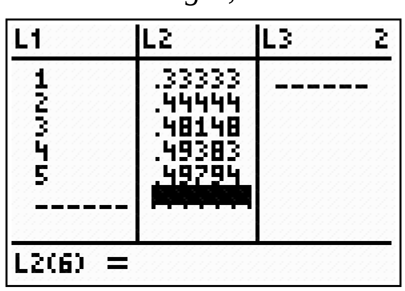
Students will label graphs with pencil.

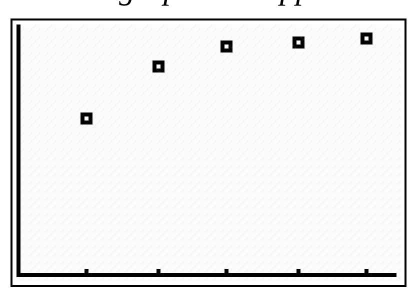
1. $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\frac{1}{243}+\ldots$

$\sum_{n=1}^{\infty}\frac{1}{3^n}$

Partial Sums:

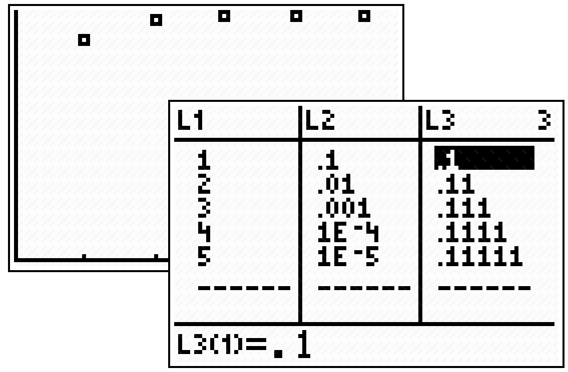
$\frac{1}{3},\frac{4}{9},\frac{13}{27},\frac{40}{81},\frac{121}{243},\ldots$





Converges to 1/2.

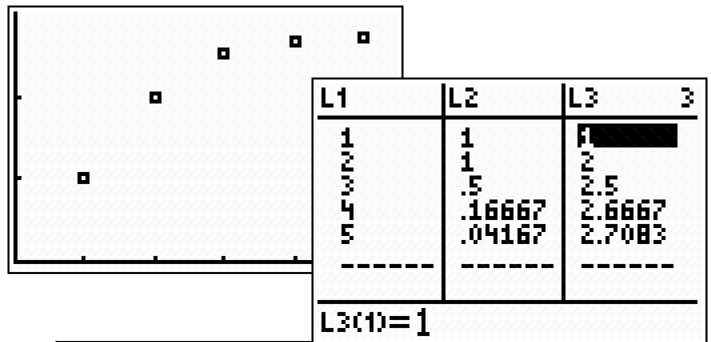
2. $\sum_{k=1}^{\infty} \frac{1}{10^k} = \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \frac{1}{100000} + \ldots$



Partial Sums:
$\frac{1}{10}, \frac{11}{100}, \frac{111}{1000}, \frac{1111}{10000}, \frac{11111}{100000}$

Converges to 1/9.

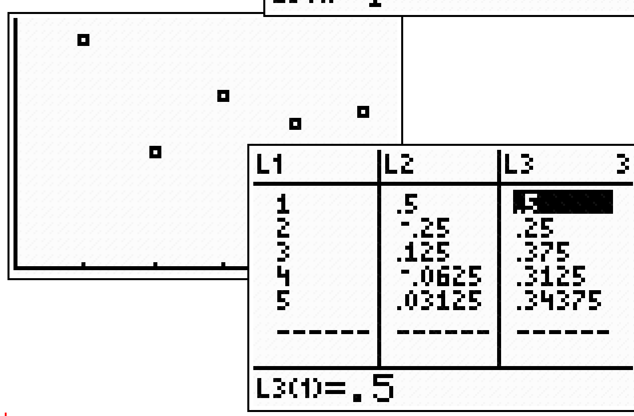
3. $\sum_{k=1}^{\infty} \frac{1}{(k-1)!} = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \ldots$



Partial Sums:
$1, 2, 2\frac{1}{2}, 2\frac{4}{6}, 2\frac{17}{24}$

Converges to e.
Look at additional partial sums.

4. $\sum_{k=1}^{\infty} (-1)^{k+1} \bullet \frac{1}{2^k} = \frac{1}{2} + \frac{-1}{4} + \frac{1}{8} + \frac{-1}{16} + \frac{1}{32} + \ldots$



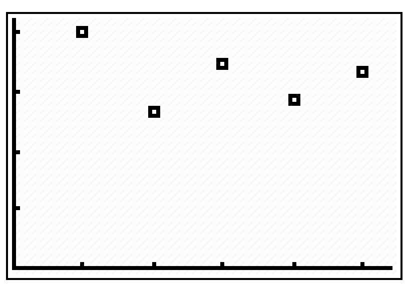
Partial Sums:
$\frac{1}{2}, \frac{1}{4}, \frac{3}{8}, \frac{5}{16}, \frac{13}{32}$

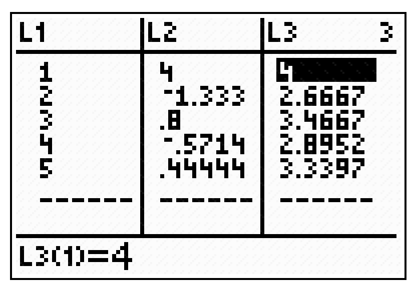
Converges to .333
Look at additional partial sums.

5. $\sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{4}{2k-1} \right) = 4 + \frac{-4}{3} + \frac{4}{5} + \frac{-4}{7} + \frac{4}{9} + \ldots$

Partial Sums:
$4, \frac{8}{3}, \frac{52}{15}, \frac{304}{105}, \frac{3156}{945}$

Converges to Pi to 4 places
- look at additional partial sums.





6. $\sum_{k=1}^{\infty} 2^k = 2 + 4 + 8 + 16 + 32 + \ldots$

Partial Sums:
$2, 6, 14, 30, 62$

Diverges