LAB: The Golden Rectangle

Name_____**ANSWER KEY**_____

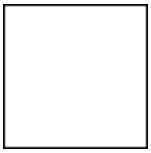
The ancient Greeks discovered that there is a specifically shaped rectangle that is most pleasing to the eye. The rectangle possessing this characteristic is called the "Golden Rectangle." Greek and Renaissance artisans used the Golden Rectangle in designing many works of art and architecture. The front of the Parthenon, in Athens, is a golden rectangle. Today you can find the Golden Rectangle in numerous everyday items. For example, credit cards and phone cards are Golden Rectangles. The ratio of the longer and shorter sides of the Golden Rectangle, called the Golden Ratio, also appears in many natural phenomena.

Construct a Golden Rectangle.

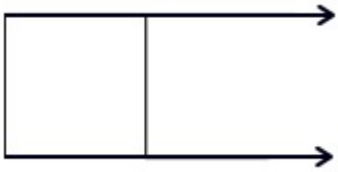
To create a rectangle with this Golden Ratio:

In a Golden Rectangle, if you cut away a square whose side is the length of the shorter side of the rectangle, you will obtain a smaller rectangle with the same shape as the original one (another Golden Rectangle). The Golden Rectangle can reproduce itself, in this manner, indefinitely.

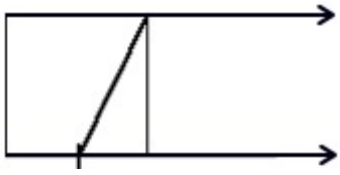
Construct a square.



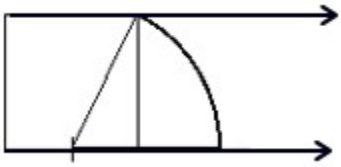
Extend two parallel sides.



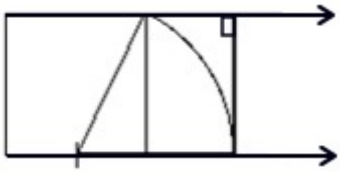
Locate the midpoint of one side of the square. Draw a segment from the midpoint to an opposite corner, as shown.



Using this segment as a radius, draw an arc between the two parallel lines.



Construct a perpendicular segment from the intersection of the arc and the bottom parallel line to the upper parallel line.

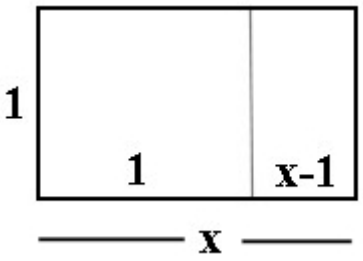


TASK: (Materials: ruler, compass, paper, scissors)

Construct a Golden Rectangle from an 8 ½ by 11 inch sheet of paper. Can you make your Golden Rectangle by making only one straight cut? Staple your Golden Rectangle to this lab sheet.

Size of answer may vary.

How is the Golden Rectangle connected to Quadratics?



Consider a Golden Rectangle as shown at the left.

Since the large rectangle and the small rectangle on the right are similar, a proportion can be established.

$$\frac{x}{1} = \frac{1}{x-1}$$

TASK: (Materials: graphing calculator)

Solve the proportion, using your knowledge of quadratics and your graphing calculator, to find the Golden Ratio. Sketch your graph at the right and show your work below.

Algebraic solution using Quadratic Formula:
Please show all work.
your answer.

$$\frac{x}{1} = \frac{1}{x-1}$$

$$x^2 - x = 1$$

$$x^2 - x - 1 = 0$$

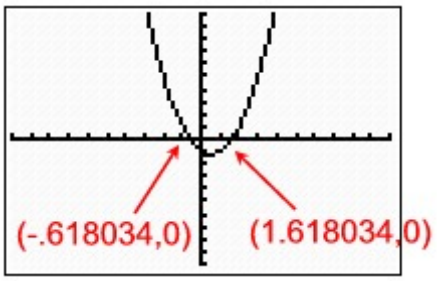
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

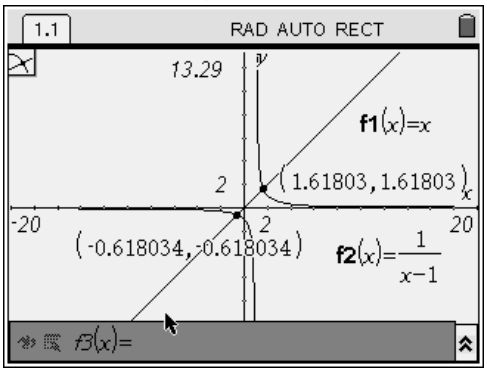
$$\frac{1+\sqrt{5}}{2} \approx 1.618033989$$

$$\frac{1-\sqrt{5}}{2} \approx -0.6180339887$$

Graphical solution using graphing calculator:
Sketch your graph and label

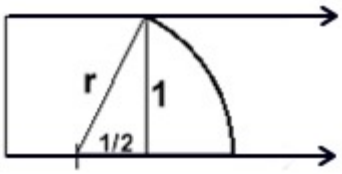


Different approaches are shown.



Let's do one last investigation in our quest for the Golden Ratio.

TASK: Using the labeling shown below, use the Pythagorean Theorem to solve for r.



$$r^2 = \left(\frac{1}{2}\right)^2 + 1^2; \quad r^2 = \frac{5}{4}; \quad r = \frac{\sqrt{5}}{2}$$

Use this value to find the Golden Ratio.

$$ratio: \quad \frac{1}{r - \frac{1}{2}} = \frac{1}{\frac{\sqrt{5}}{2} - \frac{1}{2}} = \frac{2}{\sqrt{5}-1} = 1.618033989$$

This Golden Ratio number is known as Phi (pronounced "fi") and is designated by the symbol Φ.
Phi is the ratio of the long side to the short side in a Golden Rectangle. Phi is approximately equal to 1.618