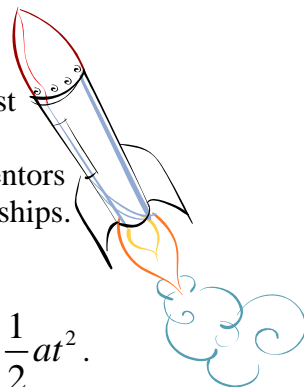


## Projectile Motion – Quadratics

October Sky

Name ANSWER KEY

The movie *October Sky* is the true story of Homer Hickam, a coal miner's son in 1950 West Virginia, who was inspired by the first Sputnik launch to take up rocketry against his father's wishes. As Homer and his friends experiment with building rockets, they experience various mishaps and are accused of setting a fire. The group of young inventors competes for a National Science Award which will provide them with college scholarships. Homer Hickam went on to become a NASA engineer.



1. When explaining why his rocket did not set the fire, Homer used the formula,  $S = \frac{1}{2}at^2$ .

a. What do the letters  $S$ ,  $a$  and  $t$  stand for in this equation?

$S =$  DISTANCE,  $a =$  ACCELERATION,  $t =$  TIME

b. This equation could also be written as  $d = 16t^2$ , which is the distance,  $d$ , in feet, that an object falls due to gravity as a function of time,  $t$ , in seconds. Explain why, in this case,  $16 = \frac{1}{2}a$ .

$a = 32$  feet per second per second      - pull of gravity  
 $16 = (1/2)32$

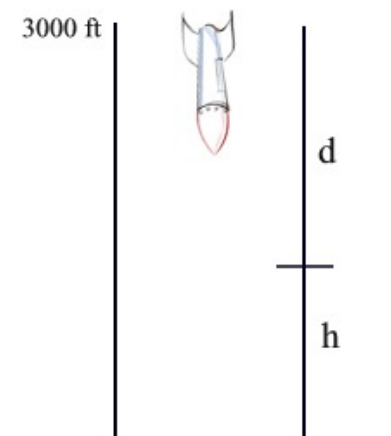
2. Homer states that his rocket fell for 14 seconds and estimates that the rocket's altitude was 3000 feet. Find the actual distance, in feet, that the rocket fell during that time.

$$d = 16t^2$$

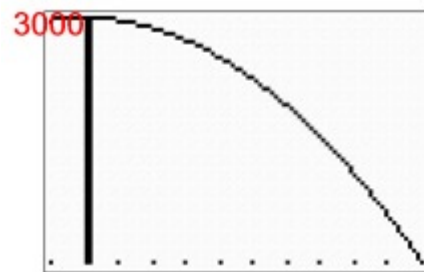
$$d = 16(14)^2$$

$$d = 3136 \text{ feet}$$

3. Using Homer's estimated altitude of 3000 feet, express the height of the falling rocket as a function of time since it began falling. Sketch the graph of this new function. Label the graph. **Students will label with pencil.**



$$h = 3000 - 16t^2$$



The vertical height of a projectile affected only by gravity near the earth's surface is modeled by a quadratic function. While the following equations do not take air resistance into account, they are, nevertheless, good models.

**METERS:** After  $t$  seconds, the height of a projectile with an initial upward velocity of  $v_0$  meters per second and an initial height  $h_0$  meters is given by  $y = h(t) = -4.9t^2 + v_0t + h_0$  meters.

**FEET:** When the initial height is measured in feet and the initial velocity is measured in feet per second, the height of the object is given by  $y = h(t) = -16t^2 + v_0t + h_0$  feet

4. According to the National Association of Rocketry, the current model rocket altitude record in the "F Altitude" event for the Ages 14-18 division is 1430 meters. From this record setting altitude, the rocket is motionless as it begins its free-fall descent back to the ground. Using the equation,  $y = h(t) = -4.9t^2 + v_0t + h_0$ , find how long it will take for the rocket to hit the ground after reaching its high point (to nearest tenth of a second).

(Hints: motionless implies  $v_0 = 0$  and hitting the ground implies  $h(t) = 0$ .)

$$1430 = 4.9t^2$$

$$t = 17.08322963$$

$$t = 17.1 \text{ seconds}$$

5. You arrive at a model rocket competition only to discover that you left your small solid fuel engines at home. In a fit of anger, you throw the engine-less rocket into the air. The rocket leaves your hand 6 feet above the ground with an initial throw velocity of 45 feet per second. Realizing that you will need the rocket for further competitions, you catch the rocket when it falls back to a height of 5 feet. For how many seconds was the rocket in the air?

Use the quadratic formula to solve for  $t$ . Express answer to the nearest tenth of a second.

$$h(t) = -16t^2 + v_0t + h_0$$

$$5 = -16t^2 + 45t + 6$$

$$0 = -16t^2 + 45t + 1$$

$$t = \frac{-45 \pm \sqrt{45^2 - 4(-16)(1)}}{2(-16)}$$

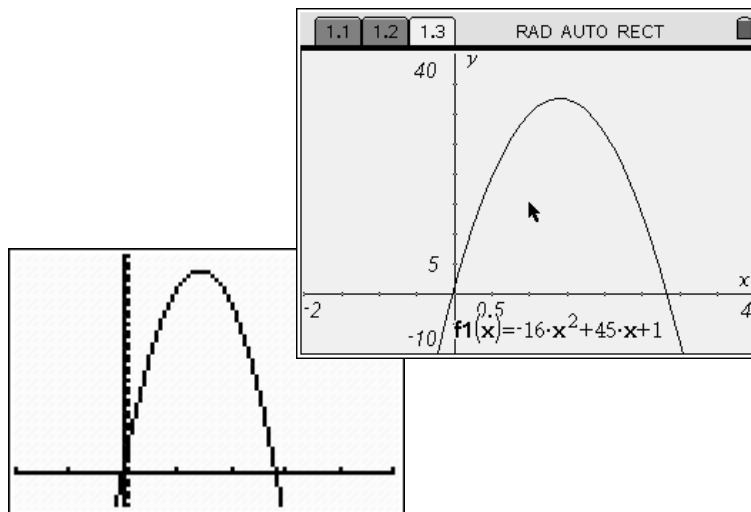
$$t = -0.0220493603$$

$$t = 2.83454936$$

$$t = 2.8 \text{ seconds}$$

Plot the graph of the rocket's height,  $h$ , versus time,  $t$ , showing the complete air-borne "flight" of the thrown rocket. Label the graph.

Students will label graph.



6. A model rocket is launched at 10 am. By 10:01 am, the rocket has reached an altitude of 15,000 feet. From point A, on ground level 25,000 feet from the launch pad, it is observed that the angle of elevation increases by  $12^\circ$  from 10:01 am to 10:02 am. Find the altitude of the rocket at 10:02 am to the *nearest tenth* of a foot. (Assume that the rocket's path is vertical.)

Let  $x$  be the missing angle at A and  $y$  be the height of the rocket at 10:01AM.

$$\tan x = \frac{15000}{25000}$$

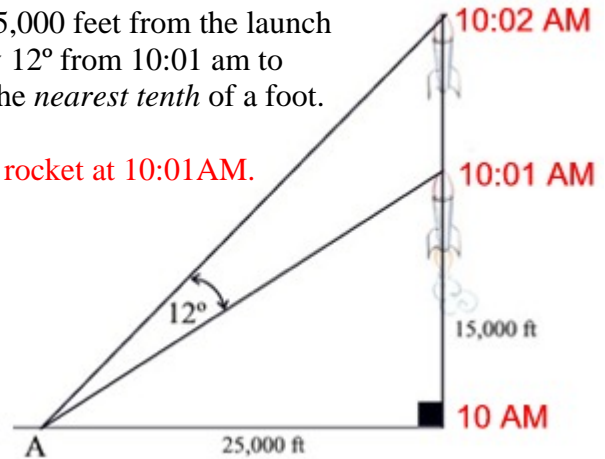
$$x = 30.96375653$$

$$x + 12 = 42.96375653$$

$$\tan 42.96375653 = \frac{y}{25000}$$

$$y = 23283.3286$$

$$y = 23,283.3 \text{ feet}$$



7. Edwards Air Force Base in California, with a concrete runway of 15,000 feet, is the touchdown site for the Space Shuttle. After touch-down, the shuttle will travel a distance,  $d$ , given by the equation

$$d = -\frac{v^2}{2r} - vt$$

where  $v$  is the shuttle's velocity,  $r$  is the shuttle's deceleration rate, and  $t$  is the number

of seconds that pass before the pilot applies the brakes. Given that the shuttle's deceleration rate is  $-15 \text{ ft/sec}^2$  and the pilot's reaction time is 0.4 seconds, what is the maximum velocity at which the shuttle can land at one end of the concrete runway without overshooting the other end?

$$d = \frac{-v^2}{2r} - vt$$

$$15000 = \frac{-v^2}{2(-15)} - v(.4)$$

$$v^2 - 12v - 450000 = 0$$

$$v = \frac{12 \pm \sqrt{(-12)^2 - 4(1)(450000)}}{2}$$

$$v = 676.8472255 \text{ ft per sec}$$

$$v = -664.8472255$$

Rounding is to be decided by the student or by the teacher:

$$v = 676.847 \text{ feet per second.}$$

8. You arrive at another model rocket competition and you have remembered to bring your small solid fuel engines. Your rocket is launched from rest and the solid fuel engine delivers a constant acceleration of 8.2 meters per second per second for 5 seconds after which the fuel is used up.

a. Using Homer's formula,  $S = \frac{1}{2}at^2$ , find the altitude of the rocket when the engine shuts down.

(Hint: This will be the initial height for part c.)

$$\begin{aligned} S &= \frac{1}{2}(8.2)5^2 \\ &= 102.5 \text{ meters} \end{aligned}$$

b. Using the formula  $v = a \cdot t$ , find the velocity (meters per sec) when the engine shuts down.

(Hint: This will be the initial velocity for part c.)

$$v = 8.2(5) = 41 \text{mps}$$

c. Find the amount of time the rocket will remain in the air after the engine shuts down.

$$h(t) = -4.9t^2 + v_0t + h_0$$

$$h(t) = -4.9t^2 + v_0t + h_0$$

$$0 = -4.9t^2 + 41t + 102.5$$

$$t = -2.014833693$$

$$t = 10.38218063$$

$$t = 10.4 \text{ seconds}$$

d. Find the TOTAL time the rocket is in the air, including the engine burn.

$$10.4 + 5 = 15.4 \text{ seconds}$$

e. Find the maximum altitude of this flight. Use your knowledge of parabolas.

Axis of symmetry

$$x = \frac{-b}{2a} = \frac{-41}{2(-4.9)} = 4.183673469$$

$$\text{max} = 188.2653061 \text{ meters}$$