**Dealing with Triangles** Name __**ANSWER KEY**________

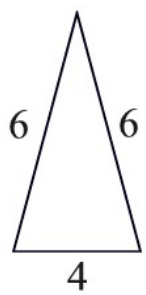
*"The Wizard of Oz"*

In the movie, *"The Wizard of Oz"*, Dorothy Gale is swept away to a magical land in a tornado and embarks upon a quest to see the Wizard who can help her return home. Accompanying her on this journey are a scarecrow (looking for brains), a tin man (looking for a heart), and a lion (looking for courage). When the wizard bestows an honorary degree of "thinkology" upon the scarecrow, the scarecrow states, "The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side."

Was the scarecrow actually stating a mathematical truth, or was he simply misstating the Pythagorean Theorem? Let's examine the scarecrow's statement.

> **Theorem????** "The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side."

**1.** Let's consider this example in relation to our questionable theorem. Examine this isosceles triangle with the given length sides. Does the theorem apply? Remember that the theorem states "ANY two sides". (Use your graphing calculator to check the validity of the statements.) State your findings.




$\sqrt{6}+\sqrt{6}\stackrel{?}{=}\sqrt{4}$ False

$\sqrt{6}+\sqrt{4}\stackrel{?}{=}\sqrt{6}$ **False**

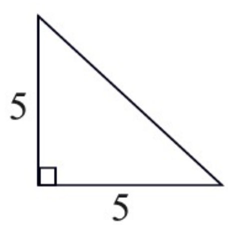
$2.449489743=\sqrt{6}$

$2=\sqrt{4}$

**2.** Create another counterexample of this theorem. Be sure that the numbers that you choose for the lengths of the sides of your triangle, do, in fact, form a triangle. Show your diagram and your work.

Answer will vary

**3.** What happens to our theorem if we add an additional restriction, such as "an isosceles RIGHT triangle"? Show whether the theorem applies to this right triangle? Hint: Find the hypotenuse first.




$hypotenuse=5\sqrt{2}$

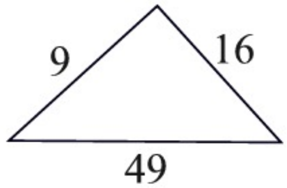
$\sqrt{5}+\sqrt{5}\neq\sqrt{5\sqrt{2}}$

$\sqrt{5}+\sqrt{5\sqrt{2}}\neq\sqrt{5}$

$\sqrt{5}=2.236067977$

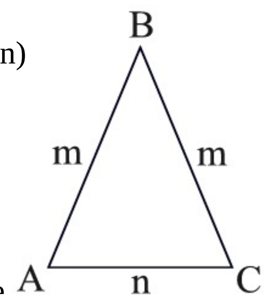
$\sqrt{5\sqrt{2}}=2.659147948$

**4.** We can attempt to generalize the scarecrow's theorem by removing the word "isosceles" and by specifying the "longest side". It now states, "The sum of the square roots of two sides of a triangle is equal to the square root of the longest side." Does the example below support this new statement? Hint: Look carefully!




**Not a triangle!!** The triangle inequality theorem does not apply to this triangle. 9 + 16 is not greater than 49.

**5.** Perhaps the scarecrow's theorem only works in the Land of Oz, but not in our Euclidean world. Prove algebraically that the scarecrow's original theorem is false.




PROOF: (An indirect proof will be used – assumption leading to a contradiction)
We will assume the scarecrow's theorem is true and see what happens.

Given $\Delta ABC$ is isosceles, with $AB = BC = m$ and $AC = n$.

Case 1: For "ANY" sides, use one leg and the base:

Assume: $\sqrt{m} + \sqrt{n} = \sqrt{m}$ [Show why this statement cannot be true in this situation.]

$$\sqrt{n} = 0$$
$$n = 0$$

$n = 0$ means the base of the triangle does not exist. A contradiction.

Case 2: For "ANY" sides, use the two legs:

Assume: $\sqrt{m} + \sqrt{m} = \sqrt{n}$ [Show why this statement cannot be true in this situation.]

$$2\sqrt{m} = \sqrt{n}$$
$$4\text{m} = n$$

$$2m > n$$
$$2m > 4m \quad \text{not true}$$

**Hints:**
1. solve for *m*

2. remember that the triangle inequality theorem states $m + m > n$

Both cases lead to a contradiction. Our assumption that the Scarecrow's theorem is true is incorrect. We have just proven that the Scarecrow's theorem is false.