

The Trouble with Tribbles

Name ANSWER KEY

Rate of Growth

Tribbles are cute, furry creatures that exist in the world of Star Trek. Being Hermaphrodites (both male and female), tribbles can produce a litter of 10 babies every 12 hours. (Assume none of them die during this initial investigation.)

1. One tribble was trapped in the grain hold of the starship Enterprise for 3 days. Science officer, Spock, proclaimed that there were 1,771,561 tribbles in the grain hold at the end of this time. Is Spock mathematically correct, or is he just mumbling TV dialog? Complete the chart below to determine if Spock's number was correct. **Powers of 11.**

After ...	The number of tribbles ...
12 hours	$1 + 10 \text{ babies} = 11 \text{ tribbles}$
24 hours	$11 + 10(11) = 121$
36 hours	$121 + 10(121) = 1331$
48 hours	$1331 + 10(1331) = 14641$
60 hours	$14641 + 10(14641) = 161051$
72 hours	$161051 + 10(161051) = 1771561$



2. Was Spock correct? Yes

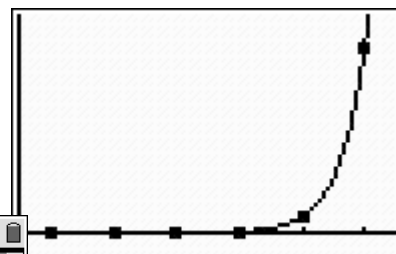
3. Using your graphing calculator, prepare a scatter plot of the data from the chart. Label your axes.

If student uses 12, 24, ...

If student uses 1, 2, ...

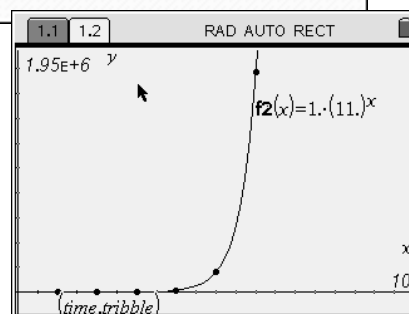
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ExpReg
y=a*b^x
a=1
b=1.22118855
r^2=1
r=1
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ExpReg
y=a*b^x
a=1
b=11
r^2=1
r=1
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A	ti...	B	tribble	C	D
					=ExpReg('tim
1	12	11	Title...	Exponentia...	
2	24	121	Reg...	a*b^x	
3	36	1331	a	1.	
4	48	14641	b	1.22119	
5	60	161051	r^2	1.	
AI	12				

A	ti...	B	tribble	C	D
					=ExpReg('tim
1	1	11	Title...	Exponentia...	
2	2	121	Reg...	a*b^x	
3	3	1331	a	1.	
4	4	14641	b	11.	
5	5	161051	r^2	1.	
DI	="Exponential Regression"				



4. Find the exponential regression equation for this data.

$y = a \cdot b^x$ answers may vary

5. What was the correlation coefficient for this model? 1

What does it tell you about the regression equation? It is a perfect fit. It is powers of 11.

6. How many tribbles were in the grain hold after 20 hours? 11 tribbles (babies every 12 hours)

A new generation of tribbles is produced every 12 hours and each generation is exactly 11 times larger than the previous generation. In addition, 11 is the sum of a parent tribble and its offspring.

A simple equation can be used to represent the rate of growth of a population:

$$r = \frac{b - d}{P}$$

r = rate of growth
 b = birth rate
 d = death rate
 P = population size.

Population size = 1 tribble

7. Using the formula above, express the rate of growth of the tribble population. $r = (10 - 0)/1 = 10$

8. If the tribble had only 4 babies in each litter, express the rate of growth of the population.

$r = (4 - 0)/1 = 4$

9. The size of the tribble population each 12 hours can be determined by multiplying the rate of growth times the population, and adding it to the current population size, where t refers to the time in **generations**.

$$P_{t+1} = rP_t + P_t = (r + 1)P_t$$

Show the use of this formula as it relates to the tribble example: $P_2 = \underline{10(11) + 1 = 121}$

10. The recursive nature of the formula in #9 makes it cumbersome for calculating population size for specific time periods, as you must compute all time periods preceding the desired time. Let's examine the numbers in search for an easier formula:

After	Generation	Population of Tribbles
0 hrs	0	1 tribble
12 hrs	1	$1 + 10 = 11 \rightarrow (10+1)^1 \cdot 1$
24 hrs	2	$11 + 10 \cdot 11 \rightarrow (10+1) \cdot 11 \rightarrow (10+1) \cdot (10+1) \rightarrow (10+1)^2 \cdot 1$
36 hrs	3	$121 + 10 \cdot 121 \rightarrow 11 (11 \cdot 10 + 11) \rightarrow 11 (11(10 + 1)) \rightarrow (10+1)((10+1)(10+1)) \rightarrow (10+1)^3 \cdot 1$

It appears from our investigation that the population size formula, where t refers to the time in generations, can be re-written as $P_t = (r + 1)^t P_0$ (P_0 is the initial population – one tribble)

Verify Spock's answer using this new formula: $P_6 = (10+1)^6 \cdot 1 = 11^6 = 1,771,561$

11. Let's compare the new formula in #10 with the exponential regression equation from the graphing calculator. Use the formulas to determine the population size in the 3rd generation = 36 hours.

$P_t = (r + 1)^t P_0$ $P_3 = (10+1)^3 \cdot 1 = 11^3 = 1,331$	Regression equation: <u>$y = 1.22118855^x$</u> $x = 36$ $y = 1331$
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